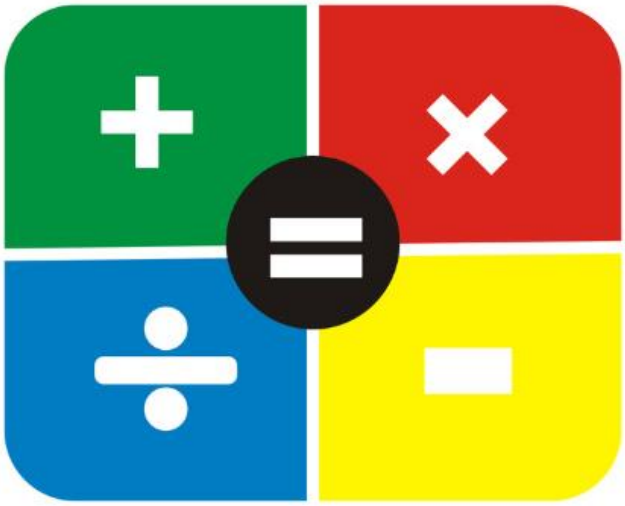
St. Michael's Church School

Calculation Policy for Mathematics



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Be the Best that you can be!

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About our Calculation Policy

The following calculation has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning calculations across our school. Please note this calculation policy is designed to build on progressively from content and methods established in the Early Years Foundation Stage.

Age Stage expectations

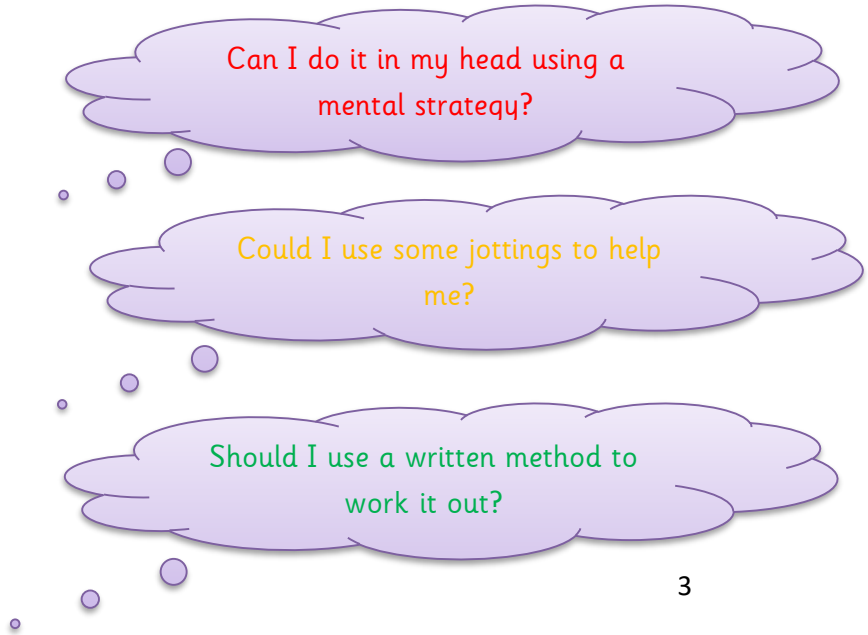
The calculation policy is organised to age-related expectations as set in the National Curriculum 2014, however it is vital that pupils are taught according to the stage that they are currently working at. Being moved onto the next level as soon as they are ready, or working at a lower stage until they are secure enough to move on.

Providing a context for calculation:

It is important that any type of calculation is given a real-life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons.

Choosing a calculation method:

Children need to be taught and encouraged to use the following processes in deciding what approach they will take to a calculation, to ensure they select the most appropriate method for the numbers involved.

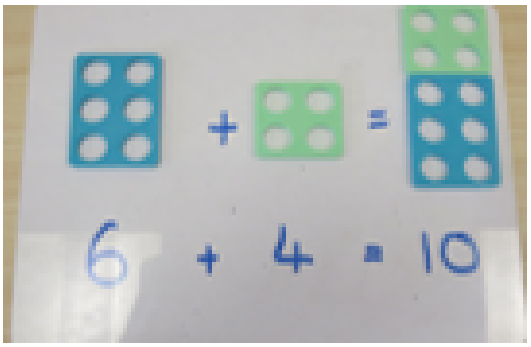


To work out a tricky calculation:
Approximate,
Calculate,
Check it!

Addition

Year 1 Add with numbers up to 20

Use Numicon to add, by counting on in ones. Encourage children to start with the larger number and count on. **For example 6 + 4 = 10** (Put 6 in your head and count on 7, 8, 9, and 10)



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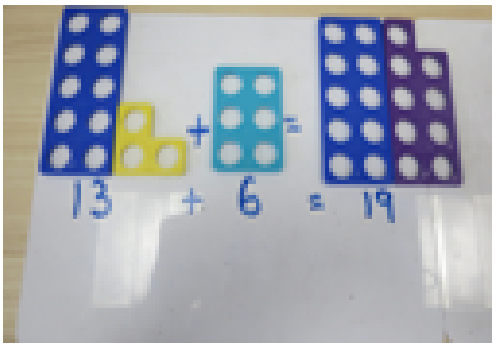


Children should:

- Have access to a wide range of counting equipment, everyday objects number tracks and number lines, and be shown numbers in different contexts.
- Read and write the addition (+) and equals (=) signs within number sentences.
- Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them. 9 + 3 = ☐

15 + 4 = ☐ 5 + 3 + 1 = ☐ ☐ + ☐ = 6

This builds on from prior learning of adding by combing two sets of objects into one group (5 cubes and 3 cubes) in Early Years. For example 8 + 5 = I be hard strategy to bridge through ten by counting on 2 then counting on 3.

Year 1 Key Vocabulary: add, more, plus, and, make, altogether, total, equal to, double, most, count on, sum of, number line, partition

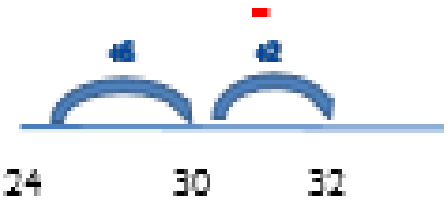
Key Skills for addition at Year 1:

- Read and write number to 100 in numerals, including 1-20 in words
- Recall bonds to 10 and 20, and additional facts **within** 20
- Count to and across 100
- Count in multiples of 1, 2, 5 and 10
- Solve simple 1-step problems involving addition, using objects, number lines, Numicon and pictorial representations.

Year 2 Add with 2-digit numbers

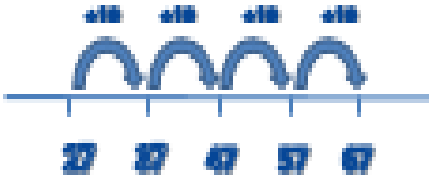
Our aim is to developing mental fluency with addition and place value involving 2-digit numbers, then establishing more formal methods

Adding 2 digit numbers and ones: 24 + 8 =

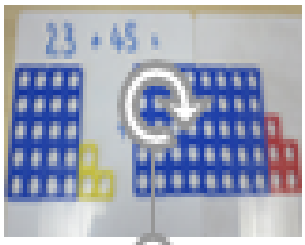


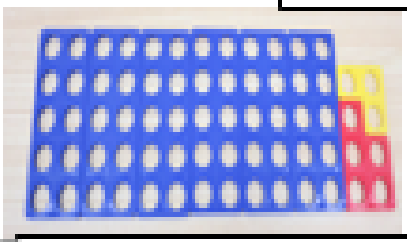
Use empty number lines, concrete equipment, hundred squares and Numicon to build confidence and fluency in mental addition

Add 2-digit numbers and tens: 27 + 40 =



Add pairs of 2-digit numbers, moving to partitioned column methods when secure adding tens and ones: 23 + 45 =

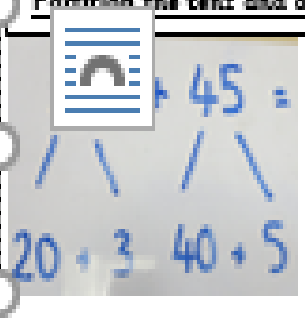




Step 1 – Only provide examples that do NOT cross the tens boundary until children are secure with the method itself.

Step 2 – Once children can add a multiple of ten to a 2-digit number mentally (80+11 they are ready for adding pairs of 2-digit numbers that do cross the ten boundary (E.g. 58 + 43 =)

Partition the tens and ones and then add together



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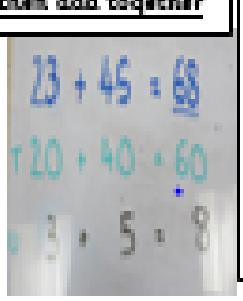


Step 3 – To partition the tens and ones and move onto the expanded addition methods with 2 and 3 digit numbers in Year 3.

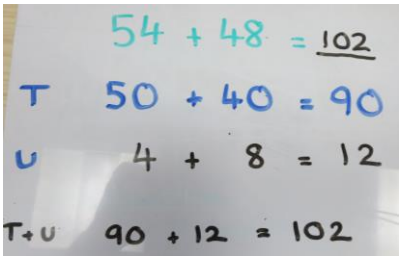
<u>Year 2 Key Vocabulary</u>: add, more, plus, and, make, altogether, total, equal to, double, most, count on, sum of, number line, partition, **tens, ones, addition, column, tens boundary**

Key Skills for addition at Year 2:

- Add a 2-digit number and ones [e.g. 34 + 5 =]
- Add a 2-digit number and tens [e.g. 23 + 30 =]
- Add pairs of 2-digit numbers [e.g. 36 + 48 =]
- Add 3 single digit numbers [e.g. 4 + 6 + 8 =]
- Show that adding can be done in any order [the commutative law]
- Recall bonds to 20 and bonds of tens to 100 [e.g. 40 + 60]
- Count in steps of 2,3 and 5 and count in tens from any number
- Understand the place value of 2-digit numbers [T and U]
- Compare and order numbers to 100 using > < and = signs
- Read and write number to at least 100 in numerals and words
- Solve problems with addition, using concrete objects, pictorial representations, involving numbers quantities and measures, and applying mental and written methods

Year 3 Add numbers with up to 3-digits

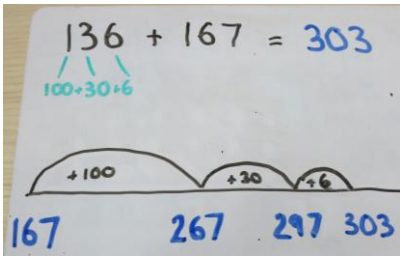
Begin with partitioning tens and ones of the smallest number and adding tens to the biggest number on a blank number line. Then add the ones to find the sum. For example 24 + 53 = 77



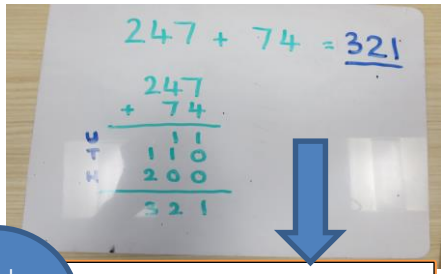
Steps

- Partition the smallest number
- Add Hundreds [H]
- Add tens [t]
- Add the ones [u]

When secure using3-digit number



Introduce the expanded column addition method:



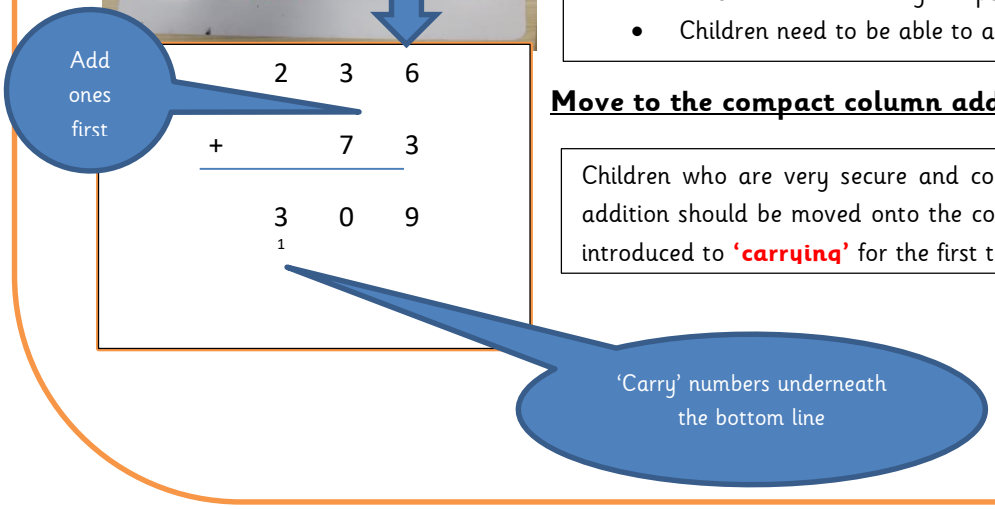
Add the ones first, in preparation for the compact method.

In order to carry out this method of addition:

- Children need to recognise the value of the hundreds, tens and ones without recording the partitioning.
- Children need to be able to add in columns.

Move to the compact column addition method with 'carrying'

Children who are very secure and confident with 3-digit expanded column addition should be moved onto the compact column addition method, being introduced to **'carrying'** for the first time.



<u>Year 3 Key Vocabulary</u>: add, more, plus, and, make, altogether, total, equal to, double, most, count on, sum of, number line, partition, tens, ones, addition, column, tens boundary, **hundreds boundary, increase, vertical, carry, expanded, compact**

Key Skills for addition at Year 3:

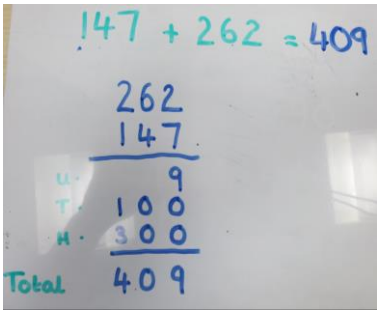
- Read and write numbers to 1000 in numerals and words
- Add 2-digit numbers mentally, incl. those exceeding 100
- Add a 3-digit number and ones mentally [e.g. 165 + 8]
- Add a 3-digit number and tens mentally [e.g. 249 + 50]
- Add a 3-digit number and hundreds mentally [381 + 400]
- Estimate answers to calculations, using inverse to check answers
- Solve problems, including missing number problems using number facts, place value, and more complex addition
- Recognise place value of each digit in 3-digit numbers [H, T, U]
- Continue to practise a wide range of mental addition strategies, i.e. number bonds, adding to nearest multiple of 10, 100, 1000 and adjusting, using near doubles, partitioning and recombining.

Year 4 Add numbers with up to 4-digits

Move from expanded addition to the compact column method, adding ones first and 'carrying' numbers **underneath** the calculation. Also include money and measure contexts.

For example: 3517 + 397 = 3914

Expanded column addition → **Compact column method**



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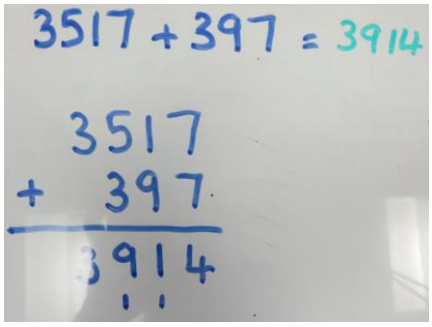
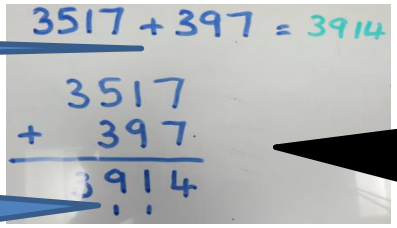


Introduce the compact column addition method by asking children to add the two numbers together using the method that they are familiar with [**expanded column addition**]. Teacher models the compact method with carrying, asking children to discuss similarities and differences and establish how it is carried out.

Use and apply this methods to money and measurement values.

Add ones first.

'Carry' numbers underneath the bottom line.



Reinforce correct place value by reminding children the actual value is **5 hundred and 3 hundred s not 5 and 3** for example.

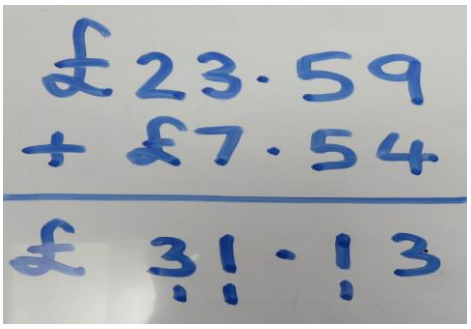
Year 4 Key Vocabulary: add, more, plus, and, make, altogether, total, equal to, double, most, count on, sum of, number line, partition, tens, ones, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact, **thousands, hundreds, digits, inverse**

Key Skills for addition at Year 4:

- Select most appropriate method: mental, jottings or written and explain why.
- Recognise the place value of each digit in a 4-digit number
- Round any number to the nearest 10, 100 or 1000
- Estimate and use the inverse operations to check answers
- Solve 2-step problems in context, deciding which operations and methods to use and why
- Find 1000 more or less than a given number
- Continue to practise a wide range of mental addition strategies i.e. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombine
- Add numbers with up to 4-digit using the formal written method of column addition
- Solve 2-step problems in contexts, deciding which operations and methods to use and why
- Estimate and use inverse operations to check the answers to a calculation.

Year 5 Add numbers with more than 4-digits

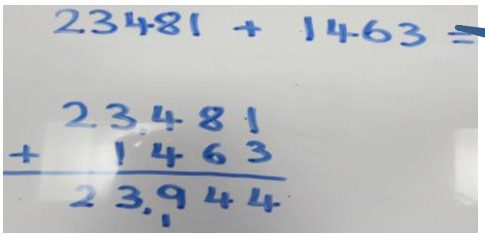
Including money, measures and decimals with different numbers of decimal places



Adding money with decimals

£23.59 + £7.54 =£31.13

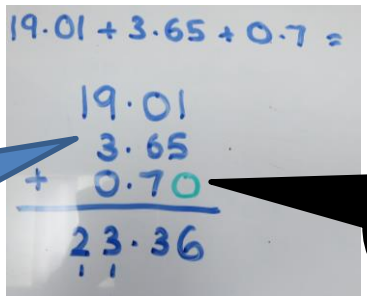
The decimal point should be aligned in the same way as the other place value columns, and must remain in the same column in the answer row.



Start with the smallest ones

Numbers should exceed 4 digits.

Children should be able to add more than two values carefully aligning place value columns.



Say '6 tenths add 7 tenths' to reinforce place vale

0.6 + 0.7 =

Empty decimal places can be filled with **zero** to show the place value in each column.

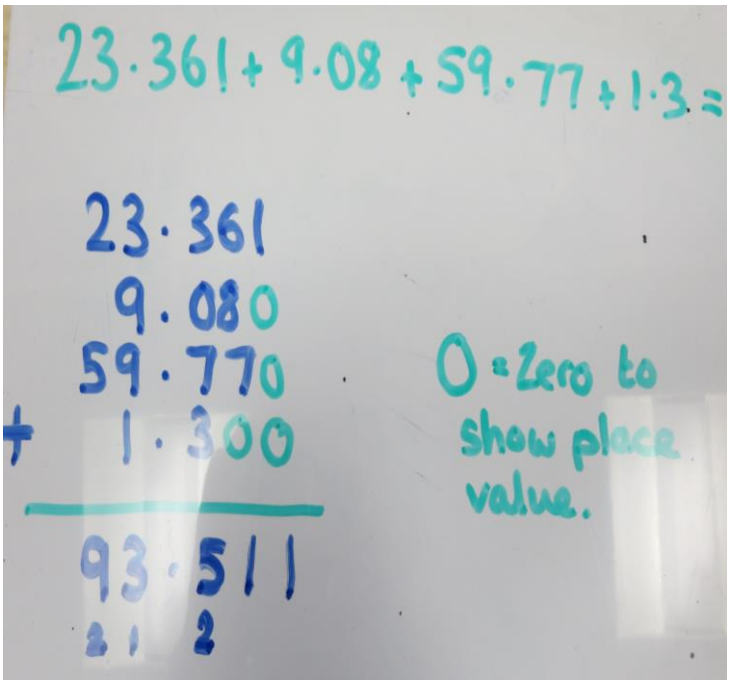
Understand the place value of tenths and hundredths and use this to align numbers with different numbers of decimals.

<u>Year 5 Key Vocabulary</u>: add, more, plus, and, make, altogether, total, equal to, double, most, count on, sum of, number line, partition, tens, ones, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact, thousands, hundreds, digits, inverse, **decimal places, decimal point, tenths, hundredths, thousandths**

Key Skills for addition at Year 5:

- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies i.e. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining
- Using rounding to check answers and accuracy
- Solve multi-step problems in context, deciding which operations and methods to use and why
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit
- Round any number up to 1,000, 000 to the nearest 10, 100, 1000, 10,000 and 100,000
- Add numbers with more than 4-digits using formal written methods of columnar addition.

Year 6 – Add several numbers of increasing complexity



Adding several numbers with different numbers of decimal places

[Including money and measures]

- Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including the answer row.
- Start with the smallest ones

Adding several numbers with more than 4 digits

		8	1	,	0	5	9
			3	,	6	6	8
		1	5	,	3	0	
		2	0	,	5	5	1
+	**1**	**2**	**0**	,	**5**	**7**	**9**
		1	1	,	1	1	

Empty decimal places should be filled with **zero** to show the place value in each column

Key Vocabulary: add, more, plus, and, make, altogether, total, equal to, double, most, count on, sum of, number line, partition, tens, ones, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths

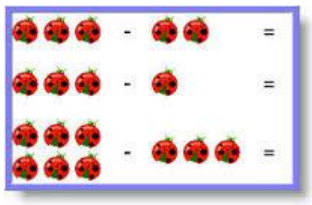
Key Skills for addition at Year 6:

- Preform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies.
- Solve multi-step problems in context, deciding which operations and methods to use and why.
- Use estimation to check answers to calculations and determine, in the context of a problem, level of accuracy.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy.
- Pupils understand how to add mentally with larger numbers and calculations of increasing complexity.

Subtraction

Year 1 Subtract from numbers up to 20

Children consolidate understanding of subtraction practically showing subtraction on bead strings, using cubes and Numicon through familiar contexts.



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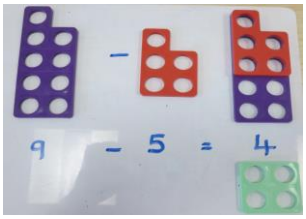
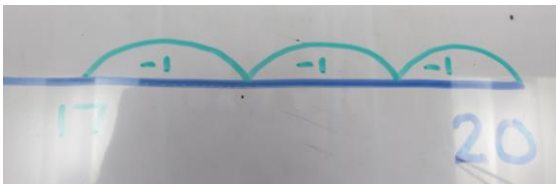


When they are secure at this children are introduced to more formal recordings using number lines as below.



Subtraction by taking away: 20 – 3 =

Count back in ones on a numbered line to take away with numbers up to 20.

Find the distance between

This will be introduced practically with the language **'find the distance between'** and **'how many more?'** in a range of familiar contexts.

7 Seven is 3 more than 4

4 I am 3 years older than my sister

Mental Subtraction: children should start recalling subtraction facts up to and within 10 and 20 and should be able to subtract zero.

Year 1 Key Vocabulary: equal to, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how many less is…?

Key Skills for subtraction at Year 1:

Given a number, say one more or less.

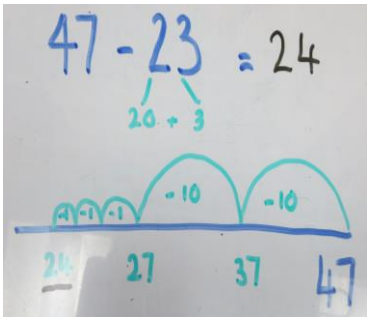
- Count to and over 100, forward and back, from any number.
- Represent and use subtraction facts to 20 and within 20.
- Subtract with 1-digit and 2-digit numbers to 20, including zero [e.g20 – 3 or 20 -14]
- Solve one-step problems that involve addition and subtraction, using concrete objects [i.e. bead strings, Numicon, objects, cubes] and missing number problems.
- Read and write numbers from 0 to 20 in numeral and words.

Year 2 Subtract with 2-digit numbers

Subtraction on a number line by counting back, aiming to develop mental subtraction skills.

This strategy with be used for:

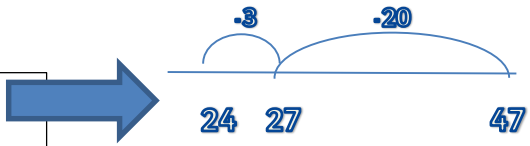
- 2-digit numbers subtract ones [**by taking away / counting back**] for example: **36 – 8 =**
- 2-digit numbers subtract tens [**by taking away / counting back**] for example: **49 – 20 =**
- Subtracting pairs of 2 digit numbers



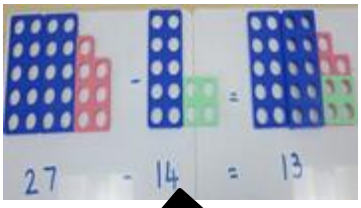
47 – 23 = 24

Partition the second number and subtract it in tens and then ones.

1. Subtract the tens
2. Then subtract the ones



Combine methods with use of 100 square and Numicon to reinforce understanding of number and place value

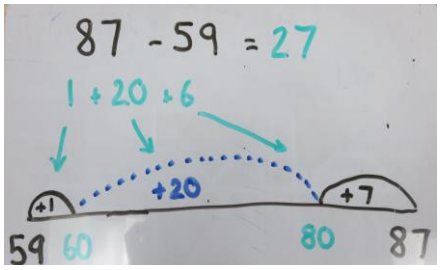


Place the 14 over the 27 and count the difference.

Teach children to use the inverse and to find the difference on a number line or using Numicon.

Find the difference

1) Add numbers to blank line
2) Find the next ten
3) Partition biggest number
4) Count in tens [60 to 80]
5) Add 3 numbers together



Mental strategy – subtract numbers close together by counting on is more efficient.

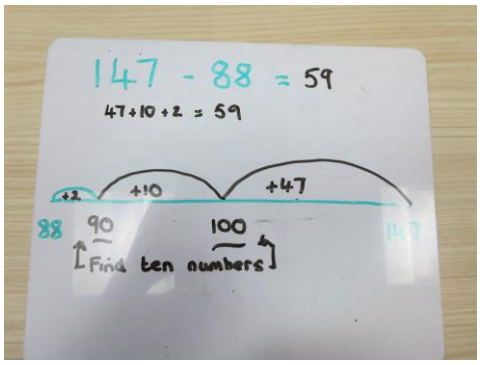
Year 2 Key Vocabulary: equal to, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how many less is...? **Difference, count on, strategy, partition, tens and ones**

Key Skills for Subtraction at Year 2:

- Recognise the place value of each digit in a two-digit number
- Recall and use subtraction facts to 20 fluently, and to derive and use related facts up to 100
- Subtract using concrete objects, pictorial representations, 100 squares and mentally, including: a 2-digit number and ones, a 2-digit number and tens, and two 2-digit numbers.
- Show that subtraction of one number from another cannot be done in any order
- Recognise and use inverse relationship between addition and subtraction, using this to check calculations and missing number problems.
- Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representations, and also apply their increasing knowledge of mental and written methods.
- Read and write numbers to at least 100 in numerals and in words.

Year 3 Subtracting with 2 and 3-digit numbers

Find the difference using 3-digit numbers



Find the difference 147 – 88 = 59

Steps

1) Add numbers to blank number line
2) Find the next 10 [88 to 90]
3) Partition the 3 digit number 147 = 100 + 47
4) Jumps in 10's [90 to 100]
5) 4) add the 3 numbers 2 + 10 + 47 = 59

Introduce partitioned column subtraction methods.

Step 1

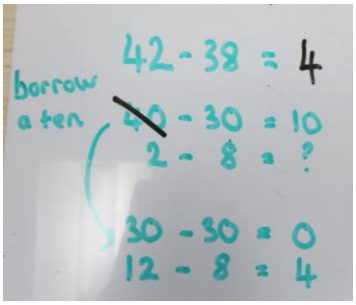
Introduce this method with examples where no **exchanging** is required.

8 9 - 3 5 = 5 4

	8	0	+	9	
-	3	0	+	5	
	5	0	+	4	

Step 2

Before you can subtract 2 from 8 you need to **exchange** a 10 for the ones. Then subtract 8 and tens.



When learning to exchange explore partitioning in different ways, so that children understand that when you exchange the value is the same i.e. 72 = 70 + 2 = 60 + 12

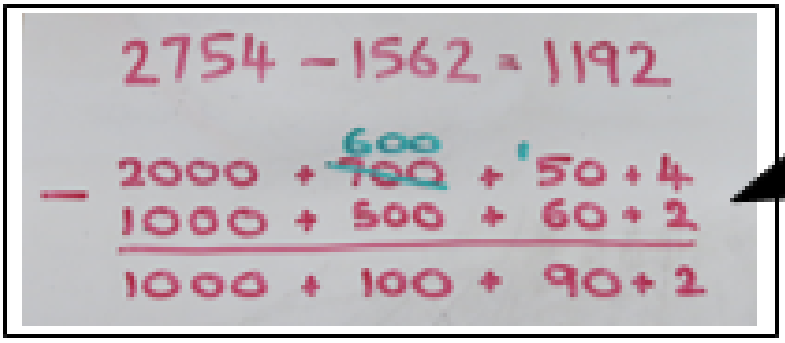
Year 3 Key Vocabulary: equal to, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how many less is...? Difference, count on, strategy, partition, tens and ones, **exchanging, decrease, hundreds, value, digit**

Key Skills for Subtraction at Year 3:

- Subtract mentally a: 3-digit number and ones, 3-digit number and tens, 3-digit number and hundreds.
- Estimate answers and use inverse operations to check.
- Solve problems, including missing number problems
- Find 10 or 100 more or less than a given number
- Recognise the place value of each digit in a 3-digit number
- Counting up differences as a mental strategy when numbers are closed together or near multiples of 10 [see examples above]
- Read and write numbers up to 1000 in numerals and words
- Practise mental subtraction, such as subtracting near multiples of 10 and adjusting [e.g. subtracting 19 or 21] and select most appropriate methods to subtract, explaining why.

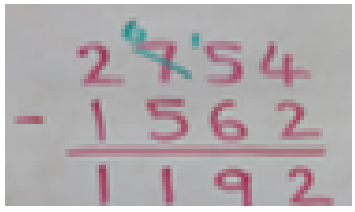
Year 4 subtract with up to 4-digit numbers

Partitioned column subtraction with 'exchanging'



As in Yr3, but moving towards more complex numbers and values. Use place value counters to reinforce exchanging.

Compact column Subtraction



To introduce the compact method, ask children to perform a subtraction calculation with the familiar partitioned column subtraction. Then display the compact version for the calculation they have done. Ask children to consider how it relates to the method they know, what is similar and what is different, to develop an understanding.

Mental Strategies

A variety of mental strategies must be taught and practised including counting on to find the difference where numbers are close together, or where it is easier to count on.

Give children plenty of opportunities to apply this to money and measures

Always encourage children to use the best method for the numbers involved – mental, counting on, counting back or written methods

Key Vocabulary: equal to, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how many less is...? Difference, count on, strategy, partition, tens and ones, exchanging, decrease, hundreds, value, digit, **inverse**

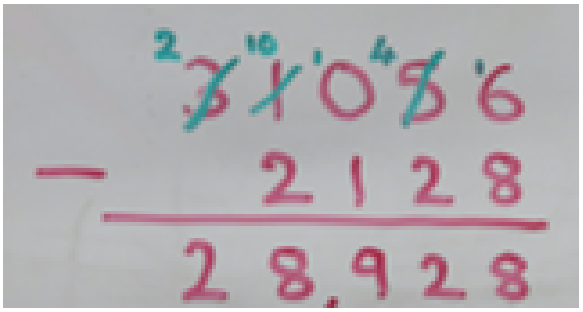
Key Skills for subtraction at Year 4:

- Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc.
- Children to select the most appropriate and efficient methods for given subtraction calculations.
- Estimate and use inverse operations to check answers.
- Solve addition and subtraction 2-steps problems, choosing which operations and methods to use and why.
- Solve simple measure and money problems involving fractions to two decimal places.
- Find 1000 more or less than a given number.
- Count backwards through zero, including negative numbers.
- Recognise place value of each digit in a 4-digit number
- Round any number to the nearest 10, 100, 1000
- Solve number and practical problems that involve the above, with increasingly large positive numbers.

Year 5 Subtract with at least 4-digit numbers

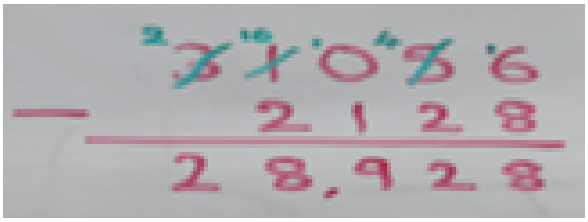
Including money, measures and decimals

Compact column subtraction – with



Children who are still not secure with number facts and place value will need to remain on the partitioned column method until they are ready for the compact method.

Subtract with decimal places, including mixtures of integers and decimals, aligning



Add a 'zero' in any empty decimal places to aid understanding of what to subtract in that column.

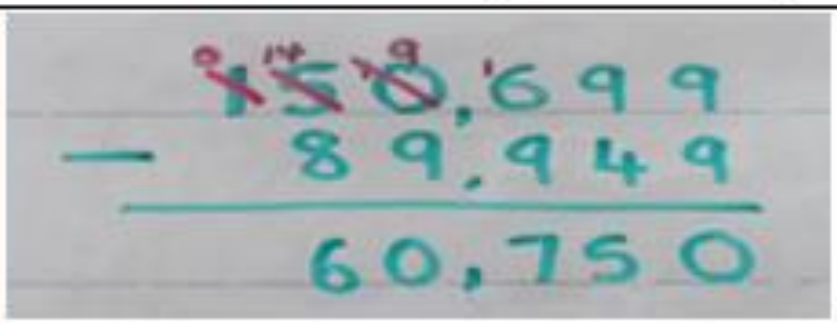
Create lots of opportunities for subtracting and finding differences with money and measures.

<u>Year 5 Key Vocabulary</u>: equal to, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how many less is...? ference, count on, strategy, partition, tens and ones, exchanging, decrease, hundreds, value, digit, inverse, **tenths, hundredths, decimal point, decimal**

Key Skills for Subtraction at Year 5:

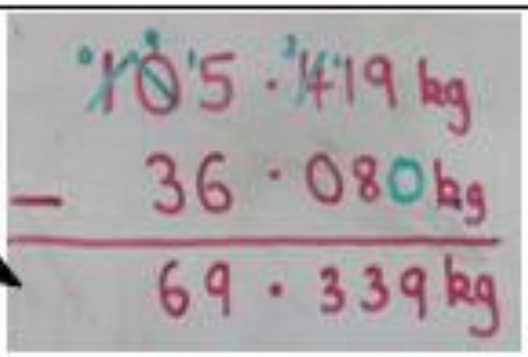
- Subtract numbers mentally with increasingly larger numbers
- Using rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy
- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write and order and compare numbers to at least 1 million and determine the value of each digit.
- Count forwards or backwards in steps of powers of 10 for any given number up to 1 million.
- Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through zero.
- Round any number up to 1 million to the nearest 10, 100, 1000, 10,000, 100,000

Year 6 – Subtracting with increasingly large and more complex numbers and decimal values



When children are secure using the compact column method, they can subtract more complex integers.

Empty decimal places can be filled with zero to show place value in each column



When children are using compact column method to subtract money and measures, include decimals with different numbers of decimal places.

Children should be able to apply their knowledge of a range of mental strategies mental recall skills, and informal and formal written method when selecting the most appropriate method to work out subtraction problems.

Year 6 Key Vocabulary: equal to, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how many less is...? difference, count on, strategy, partition, tens and ones, exchanging, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal

Key Skills for subtraction at Year 6:

- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy
- Use negative numbers in context, and calculate intervals across zero
- Children to utilise and consider a range of mental subtraction strategies, jottings, and written methods before choosing how to calculate.

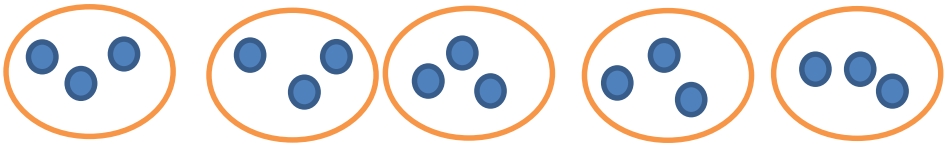
Multiplication

Year 1 multiply with concrete objects, Numicon, arrays and pictorial representations.

How many legs with 3 teddies have?

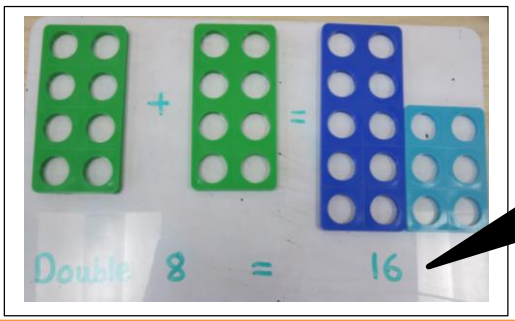
2 + 2 + 2 = 6

There are 3 sweets in one bag. How many sweets are there in 5 bags altogether?



3+3+3+3+3 = 15

- Give children experiences of counting equal groups of objects in 2's, 5's and 10's.
- Present practical problem solving activities involving counting equal sets or groups, as above



To learning doubling is the same as multiplying by 2.

Year 1 Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count

Key Skills for Multiplication at Year 1:

- Count in multiplies of 2, 5 and 10
- Solve one step problems involving multiplication, by calculating the answer using concrete objects, Numicon, pictorial representations and arrays with the support of the teacher.
- Make connections between arrays, number patterns, and counting in two's, fives and tens.
- Begin to understand doubling using concrete objects, Numicon and pictorial representations

Year 2 multiply using arrays and repeated addition [using at 2's, 5's and 10's]

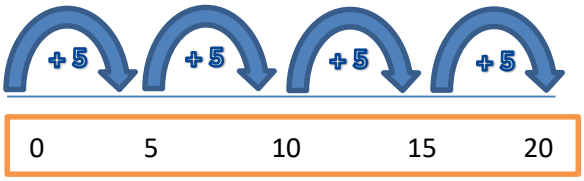
Use repeated addition on a number line: **4 x 5 = 20**

Starting from zero, make equal jumps on a number line to

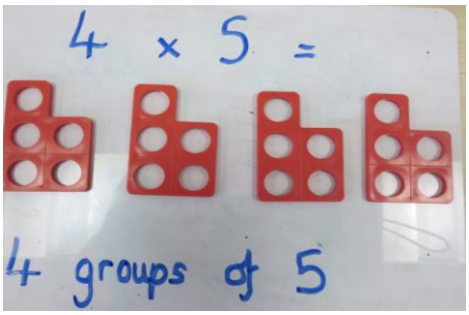
Work out multiplication facts and write multiplication

Statements using **x and =** signs.

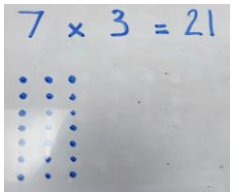
4 lots of 5



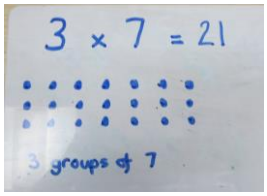
Use arrays:



is the same as



Can be done in any order



Use mental recall – Children should be able to recall multiplication facts or 2, 3, 4, 5 and 10 times tables.

Year 2 Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count **multiplied by, repeated addition, column , row, commutative, sets of, equal groups, times as big as, once, twice, three times…**

Key Skills for Multiplication at Year 2:

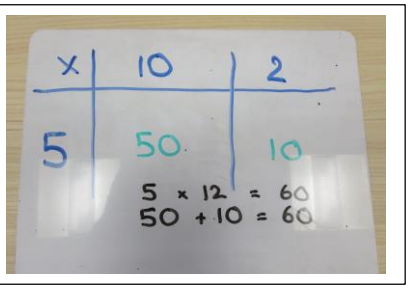
- Count in steps of 2, 3 and 5 from zero, and in 10's from any number
- Recall and use multiplication facts from the **2 5 and 10** multiplication tables, including recognising odds and evens.
- Write and calculate number statements using the x and = signs.
- Show that multiplication can be done in any order [commutative]
- Solve a range of problems involving multiplication using concrete objects, Numicon, arrays, repeated addition, mental methods and multiplication facts.
- Pupils use a variety of language to discuss and describe multiplication.

Year 3 multiply 2-digit numbers by a single number

Introduce the grid method for multiplying 2-digit numbers by single digits:

E.g. **23 x 8 = 184**

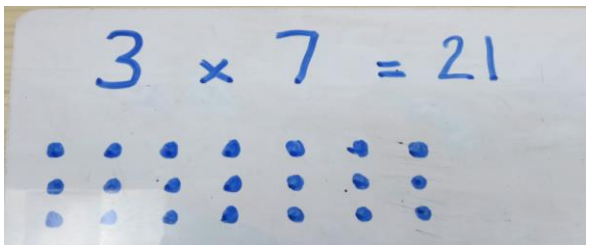
X	20	3
8	160	24



Introduce the grid method with children physically making an array to represent the calculation [e.g. make 5 lots of 10 with 10's Numicon and 1's Numicon]. Then translate this to the grid method.

To do this, children must be able to:

- Partition numbers into tens and ones
- Multiply multiples of ten by a single digit [20 x 4] using their knowledge of multiplication facts and place value.
- Recall and work out multiplication facts in the 2, 3, 4, 5, 6, 8, and 10 times tables.
- Work out multiplication facts by known by repeated addition or other taught mental strategies [e.g. commutative law, working out near multiplies and adjusting, using doubling etc.]. Strategies to support this are repeated additional using number line, bead bars and arrays.



Year 3 Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count multiplied by, repeated addition, column , row, commutative, sets of, equal groups, times as big as, once, twice, three times… **partition, grid method, multiple, product, tens, ones, value**

Key Skills for Multiplication at Year 3:

- Recall and use multiplication facts for the **2, 3, 4, 5 8 and 10** multiplication tables, and multiply multiples of 10.
- Write and calculate number statements using the multiplication tables they know, including 2-digit x single digit, drawing upon mental methods, and progressing to reliable written methods.
- Solve multiplication problems, including missing number problems.
- Developmental strategies using commutativity [e.g. 4 x 12 x 5 = 4 x 5 x 12 x 20 x 12 = 240]
- Solve simple problems in context, deciding which operations and methods to use.
- Develop efficient mental methods to solve a range of problems e.g. using commutativity [4 x 12 x 5 = 4 x 5 x 12 = 20 x 12 = 240] and missing number problems? x 5 = 20 3 x a = 18 a x b = 32

Year 4 Multiply 2 and 3-digit numbers by a single digit

Using all multiplication tables up to 12 x 12

Developing the grid method: 136 x 5 = 680

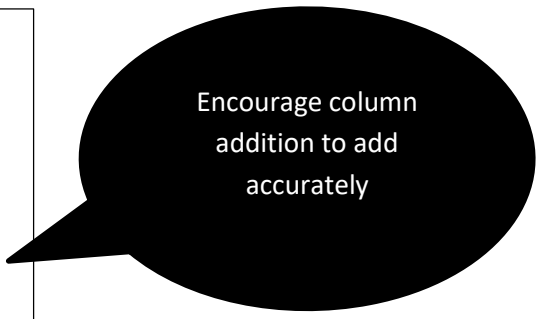
x	100	30	6
5	500	150	30

500

150

+ 30

680



Move onto short multiplication [see yr5] if and when children are confident and accurate multiplying 2 and 3-digit numbers by a single digit this way and are already confident in **carrying** for written addition.

Children should be able to:

- Approximate before they calculate, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer.

  For example – 346 x 9 is approximately 350 x 10 = 3500
- Record an approximation to check their final answer against.
- Multiply multiplies of ten and on hundred by a single digit, using their multiplication table knowledge.

Recall all times tables up to 12 x 12

Year 4 Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count multiplied by, repeated addition, column , row, commutative, sets of, equal groups, times as big as, once, twice, three times… partition, grid method, multiple, product, tens, ones, value**, inverse**

Key Skills for multiplication at Year 4:

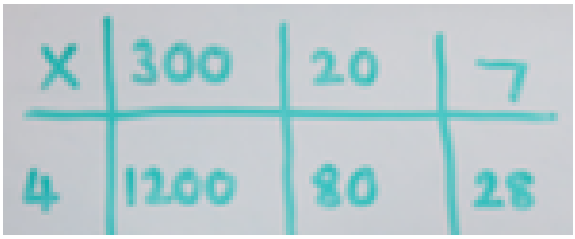
- Count in multiples of 6, 7, 9, 25 and 1000
- Recall multiplication facts for **all multiplication tables up to 12 x 12**
- Recognise place value of digits in up to 4-digit numbers
- Use place value, known facts and derived facts to multiply mentally e.g. multiply by 1, 10, 100 by 0, or to multiply 3 numbers.
- Use commutativity and other strategies mentally e.g. 3 x 6 = 6 x 3 2 x 6 x 5 = 10 x 6 39 x 7 = 30 x 7 + 9 x 7
- Solve problems with increasingly complex multiplications in a range of contexts.
- Count in multiples of 6,7,9, 25 and 1000
- Recognise the place value of each digit in a four-digit number [thousand hundreds, tens and ones]

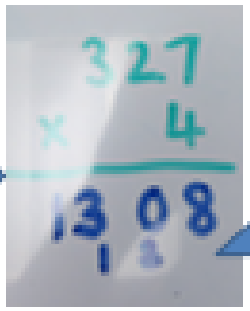
Year 5 Multiply up to 4-digits by 1 or 2 digits

Introduce column multiplication

- Introduce by comparing a grid method calculation to a short multiplication method, to see how the steps are related, but notice how there are less steps involved in the column method.
- Children need to be taught to approximate first e.g. for 72 x 38, they will use rounding: 72 x 38 is approximately 70 x 40 = 2800, and uses the approximation to check the reasonableness of their answers against.

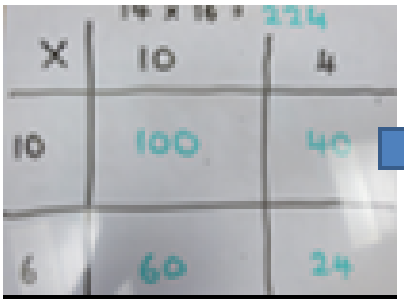
Single multiplication for multiplying by a single digit





Compare similarities and differences in your teaching

Introduce long multiplication for multiplting by 2 digits



		1	8
x		1	3
		5	4
	1	8	0
	2	3	4

18 x 3 on first row

8 x 3 = 24 carries the 2 for twenty then 10 x 3

18 x 10 on the 2nd row. Put a zero in one's first then say 8 x1 and 1x1

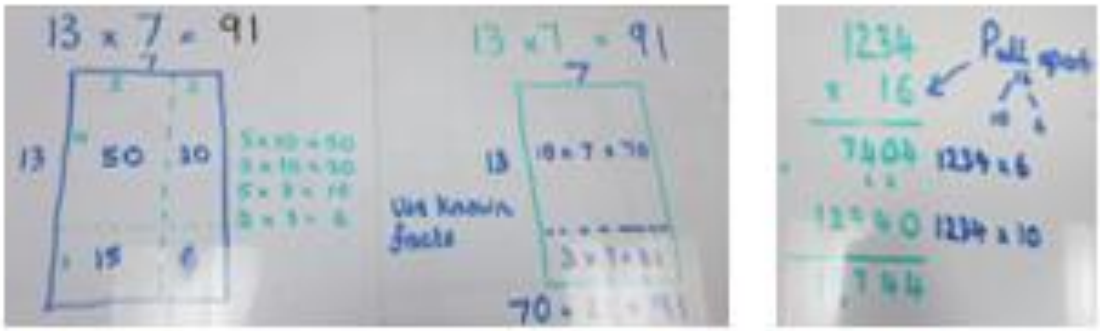
Year 5 Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count multiplied by, repeated addition, column , row, commutative, sets of, equal groups, times as big as, once, twice, three times... partition, grid method, multiple, product, tens, ones, value, inverse, **square, factor, integer, decimal, short/long multiplication, carry**

Key Skills for Multiplication at Year 5:

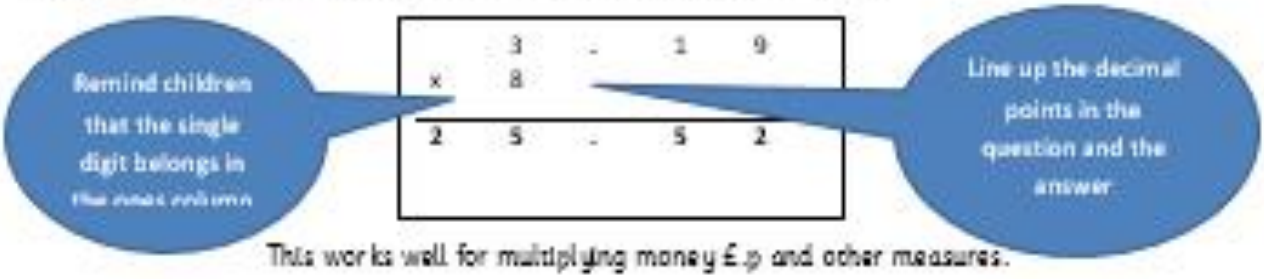
- Identify multiples and factors, using knowledge of multiplication tables to 12 x 12
- Solve problems where larger numbers are decomposed into their factors
- Multiply and divide integers and decimals by 10, 100 and 1000
- Recognise and use square and cube numbers and their notation
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately

Year 6 – Add several numbers of increasing complexity

Short and long multiplication as in Year 5, and multiply decimals with up to 2 decimal points by a single digit.



This is the grid method but linked to area to show the commutative law.



This works well for multiplying money £.p and other measures.

Children will be able to:

- Use rounding and place value to make approximations before calculating and use these to check answers against.
- Use **short multiplication** [see Y5] with more than 4-digit numbers by a single digit, to multiply mone and measure and to multiply decimales with up to 2 d.p by single digit.
- Use **long multiplication** [see Y5] to multiply numbers with at least 4 digits by a 2 digit number.

Year 6 Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count multiplied by, repeated addition, column , row, commutative, sets of, equal groups, times as big as, once, twice, three times… partition, grid method, multiple, product, tens, ones, value, inverse, square, factor, integer, decimal, short/long multiplication, carry, **tenths, hundredths, decimal**

Key Skills for Multiplication at Year 6:

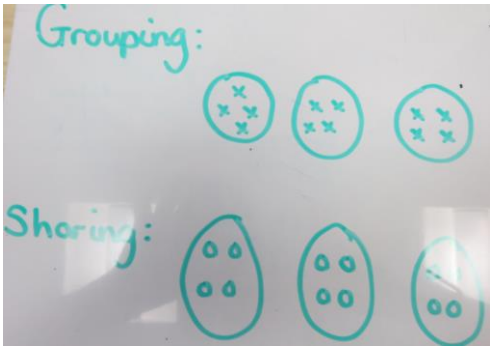
- Recall multiplication facts for all times tables up to 12 x 12 as Yr4 and Yr5
- Multiply multi-digit numbers, up to 4-digit x 2-digit numbers using long multiplication
- Perform mental calculations with mixed operations and large numbers

Division

Year 1 Group and share small quantities

Using objects, diagrams, Numicon and pictorial representations to solve problems involving **both grouping and sharing**

How many groups of 4 can be made with 12 stars? = 3



12 shared between 3 is 4

Example division problem in a familiar context:

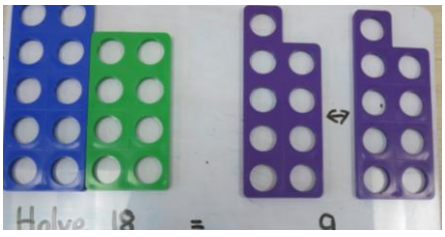
There are 6 pupils on this table and there are 18 pieces of fruit to share between us. If we share them equally, how many will we each get?

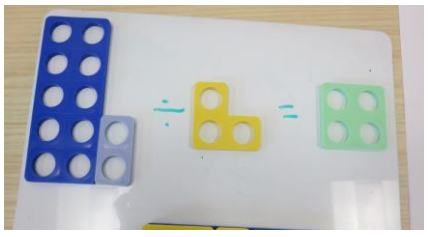
Can they work it out and give a division statement?

18 shared between 6 people gives you 3 each.

Children should:

- Use lots of practical apparatus, arrays and picture representations.
- Be taught to understand the differences between grouping objects [**how many groups of 2 can you make?**] And sharing [**share these sweets between 2 people?**]
- Be able to count in multiples of 2's , 5's and 10's
- Find **half of a group** of objects by sharing into 2 equal groups





Year 1 Key Vocabulary: share, share equally, one each, two each, group, groups of, lots of, array, sharing pots

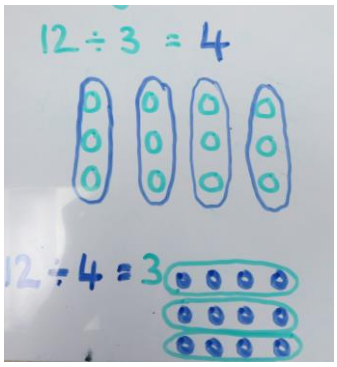
Key Skills for division at Year 1:

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, Numicon, pictorial representations, arrays with support of the teacher.
- Through grouping and sharing small quantities, pupils begin to understand, division and finding simple fractions of objects, numbers and quantities.
- They make connections between arrays, number patterns, and counting in two's, fives, and tens.

Year 2 Group and share, using the - and = sign

Use objects, arrays, diagrams and pictorial representations, and grouping on a number line.

Arrays:



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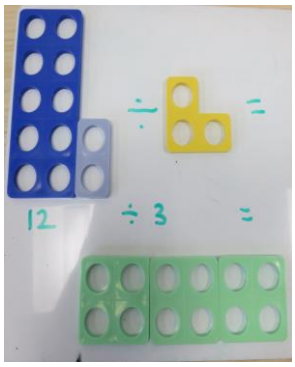

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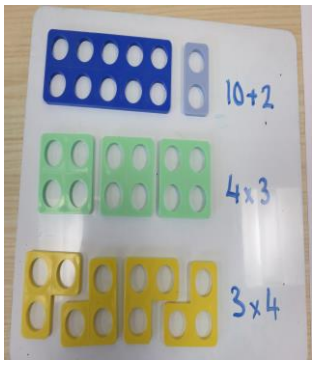
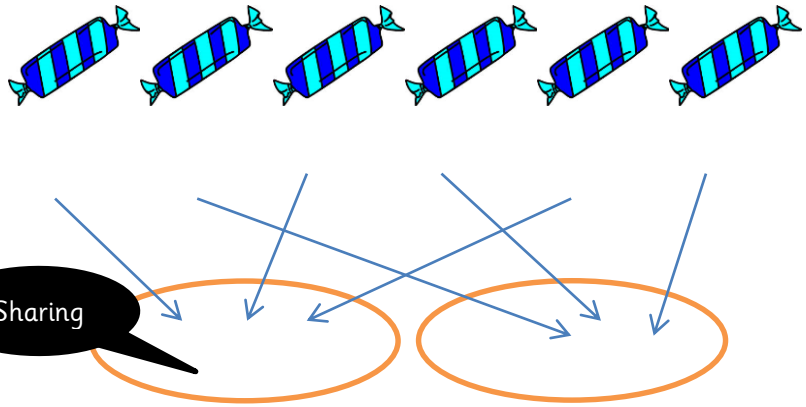


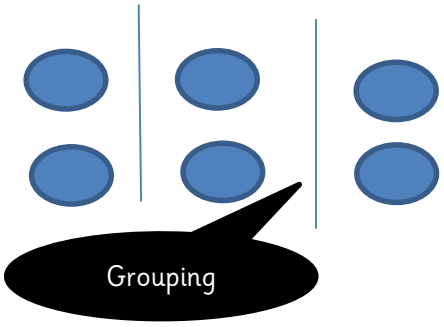
This represents 12 divided by 3. Posed as how many groups of 3 are in 12?

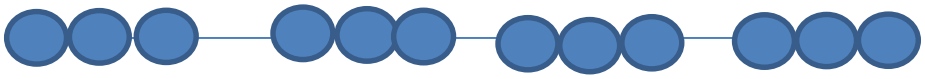
Children should also show that the same array can represent 12 divided by 4 = 3

As show opposite.

Know and understand sharing and grouping: 6 sweets shared between 2 people. How may do they get?







Grouping using a number line: Group from zero in equal jumps of the divisor to find out how many groups in __ in __ ? Children could use bead strings or practical apparatus [Numicon] to work out problems. A Cd cost £3. How many Cd's can I buy with £12? **This develops understanding of division as grouping.**

Year 2 Key Vocabulary: share, share equally, one each, two each, group, groups of, lots of, array, sharing pots **divide, divided by, divided into, grouping, number line, left over**

Key Skills for division at Year 2:

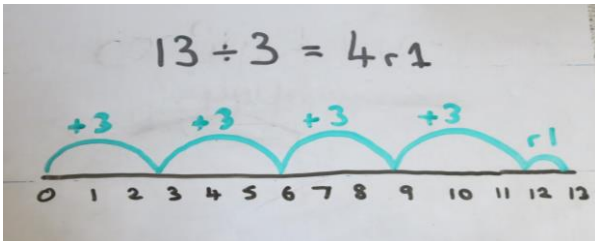
- Count in steps of 2, 3 and 5 from 0
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using x – and = signs.
- Show that multiplication of two numbers can be done in any order [commutative] and divisions of one number by another cannot.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, Numicon, and multiplication and division facts, including problems in context

Year 3 Divide 2-digit number by a single digit

[Where there is no remainder in the final answer]

Grouping on a number line:

13 divided by 3 = 4 r 1

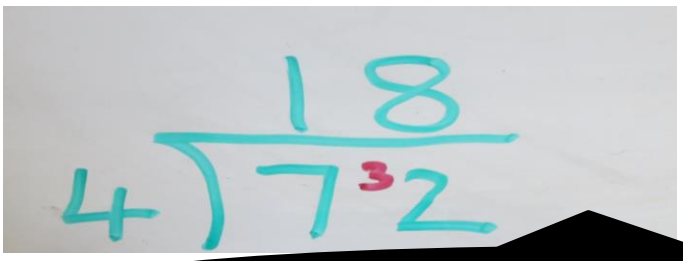


Step 1: Children continue to work out unknown division facts by grouping on a number line from zero. T**hey are also now taught the concept of remainders**, as in the example. This should be introduced practically and with arrays, as well as the number line. Children should work towards calculating some basic division facts with remainders mentally for 2's 3's 4's 5's 8's and 10's, ready for carrying remainders across within the short division method

Short division: with no remainder

Step 2: Once children are secure with division as grouping and demonstrate this using number lines, arrays. Then short division for larger 2-digit numbers should be introduced, initially with carefully selected **examples with no remainders.** Start by introducing them layout of short division by comparing it to an array.

Short division: no remainders in final answer



Step 3: When children understand of remainders, and also confident at **short division**, they can be taught how to use the method with **remainders** occurring within the calculation [e.g. 94 divide 4] and be taught to **'carry'** the remainder onto the next digit.

Only teach when children can calculate remainders

Year 3 Key Vocabulary: share, share equally, one each, two each, group, groups of, lots of, array, sharing pots divide, divided by, divided into, grouping, number line, left over, **inverse, short division, carry, remainder, multiple**

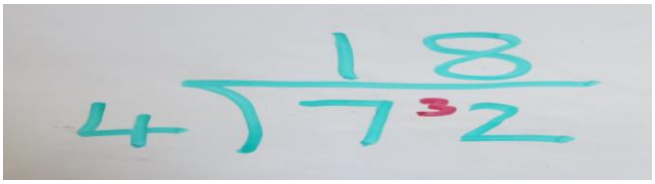
Key Skills for addition at Year 3:

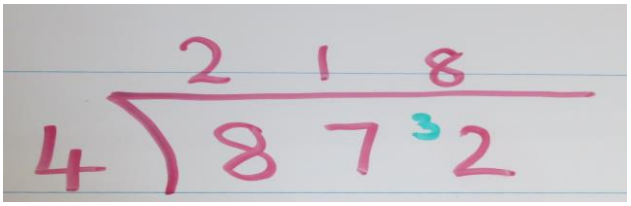
- Recall and use multiplication and division facts for 2, 3, 4, 5, 8 and 10 multiplication tables [through doubling , connect the 2, 4 and 8's]
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers time's one-digit numbers, using mental and progressing to formal written methods.
- Solve problems, in contexts, and including missing number problems, involving multiplication and divisions.
- Pupils develop efficient mental methods , for example, using multiplication and division facts [e.g. using 3 x 2 = 6 6 divided by 3 = 2] to derive related facts [30 x 2 = 60 so 60 divided by 3 = 20]
- Pupils develop reliable written methods for division, starting with calculations of 2-digit numbers by 1-digit numbers and progress to the formal written method

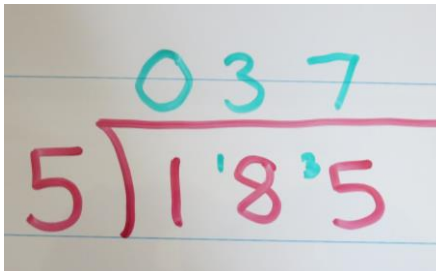
Year 4 divide up to 3-digti numbers by a single digit number

Continuing to develop short division:

Short division only introduce if secure with calculating

Step 1:	Children must be secure with the process of short division for dividing 2-digit numbers by single digit [those that do not result in a final remainder – See Yr3] but must understand how to calculate **remainders**, using this to '**carry remainders'** within the calculation process.
	

Step 2:	Children need to move onto dividing numbers with up to 3-digits by a single digit; however problems and calculations provided should not result in a final answer with remainders at this stage. Children who exceed this expectation may progress to Y5 level.
	



When the answer for the first column is zero [1 divided by 5] children could initially write a zero above to acknowledge its place and must always carry the number [1] over to the next digit as a remainder.

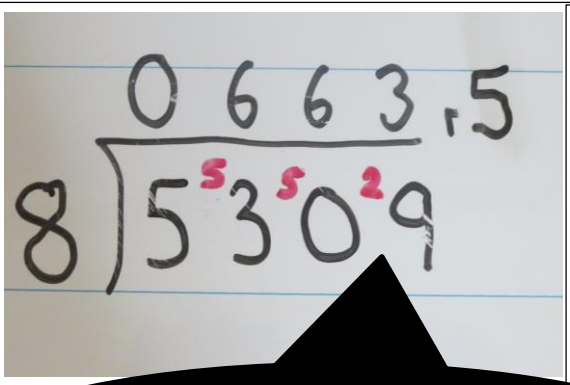
Year 4 Key Vocabulary: share, share equally, one each, two each, group, groups of, lots of, array, sharing pots divide, divided by, divided into, grouping, number line, left over, inverse, short division, carry, remainder, multiple, **divisible by, factor**

Key Skills for Multiplication at Year 4:

- Recall multiplication and division facts for all numbers up to 12 x 12
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.
- Pupils to practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number
- Pupils practise mental methods and extend this to 3-digti numbers to derive facts for example 200 x 3 = 600 so 600 divided by 3 = 200
- Pupils solve two-step problems in context, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

Year 5 Divide up to 4 digits by a single digit including those with remainders

Short division, including remainders answers:



Now children are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where children consider the meaning of the remainder and **how** to express it, i.e. as a fraction, a decimal, or as a rounded number or value depending upon the context of the problem.

The answer to 5309 divided by 8 could be expressed as **663 and 5 eighths**, **663 r5, as a decimal**, or rounded as appropriate to the problem involved.

See Yr6 for how to continue the short division to give a decimal answer for children who are

If children are confident and accurate:

- Introduce long division for children who are ready to divide any number by 2-digit number [e.g. 2678 divided by 19]. This is a Year 6 expectation see Yr6

Year 5 Key Vocabulary: share, share equally, one each, two each, group, groups of, lots of, array, sharing pots divide, divided by, divided into, grouping, number line, left over, inverse, short division, carry, remainder, multiple, divisible by, factor, **inverse, quotient, prime numbers, composite number [non-prime]**

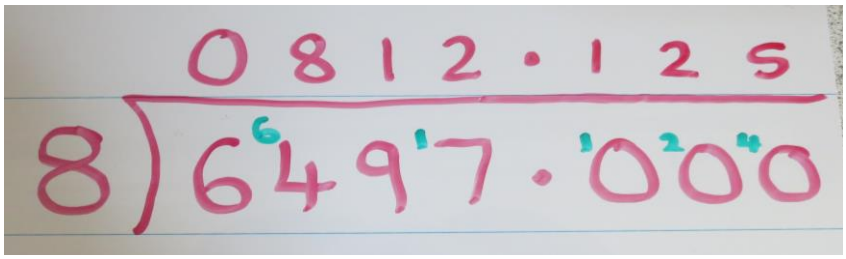
Key Skills for division at Year 5:

- Recall multiplication and division facts for all numbers up to 12 x 12 [as in Year 4]
- Multiply and divide numbers mentally, drawing upon known facts.
- Identify multiples and factors, including finding all factors pairs of a number, and common factors of two numbers.
- Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
- Use the vocabulary of prime numbers, prime factors and composite [non-prime] numbers.
- Work out whether a number up to 100 is prime, and recall prime numbers to 19.
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.
- Use multiplication and division as inverses.
- Solve problems involving combinations of all 4 operations, including understanding of the equal sign, and including division for scaling by different fractions and problems involving simple ratios.

Year 6 – Divide at least 4 digits by both single-digit and 2-digit numbers

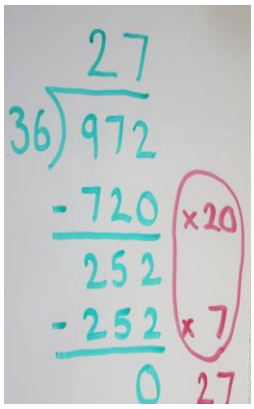
[Including decimal numbers and quantities]

Short division, for dividing by a single digit: **6497 divided by 8**



Calculating a decimal remainder: Rather than expressing the remainder as r1. A decimal point is added after the units because there is still a remainder, and the one remainder is carried onto zeros after the decimal point.

Children should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. **Real life problems solving** contexts need to be the starting point, where children have to consider the most appropriate way to express the remainder.



Introduce long division by chunking for dividing by 2 digits.

- Find out 'How many 36's are in 972? By subtracting 'chunks of 36, until zero is reached [or until there is a remainder].
- Teach children to write a 'useful list' first at the side that will help them decide what chunks to use e.g.
- Introduce the method in a simple way Can we use x10, x100 then encourage more efficient chunks to get answer quickly [x20 x5] and expand on their list.

Useful list

1 x 36 = 36

10 x = 360

100 x 3600

Year 6 Key Vocabulary: share, share equally, one each, two each, group, groups of, lots of, array, sharing pots divide, divided by, divided into, grouping, number line, left over, inverse, short division, carry, remainder, multiple, divisible by, factor, inverse, quotient, prime numbers, composite number [non-prime]**, common factor**

Key Skills for division at Year 6:

- Recall and use multiplication and division facts for all numbers 12 x 12 for more complex calculations
- Divide numbers up to 4 digits by two-digit whole numbers using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate.
- Preform mental calculations, including with mixed operations and large numbers
- Identify common factors, common multiples and prime numbers
- Solve problems involving all 4 operations
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem
- Use written division methods in cases where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specific degrees of accuracy.

Attachment: Pictorial representation:

In maths, a **bar model** is a pictorial representation of a problem or concept where bars or boxes are used to represent the known and unknown quantities. Bar models are most often used to solve number problems with the four operations – addition, subtraction, multiplication and division.

In word problems, bar models help children decide which operations to use or visualise problems.

The bar model is central to maths mastery, the pictorial stage in the concrete pictorial abstract (CPA) approach to learning.

Key stage 1

Addition

Pupils in Reception and Year 1 will routinely come across calculations such as 4+3.

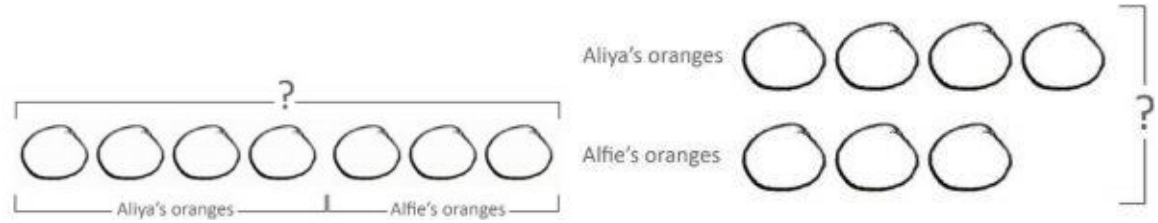
Often, these calculations will be presented as word problems: Aliya has 4 oranges. Alfie has 3 oranges. How many oranges are there altogether? With addition, subtraction and multiplication, to help children fully understand later stages of bar modelling, it is crucial they begin with concrete representations.

There are 2 models that can be used to represent addition:

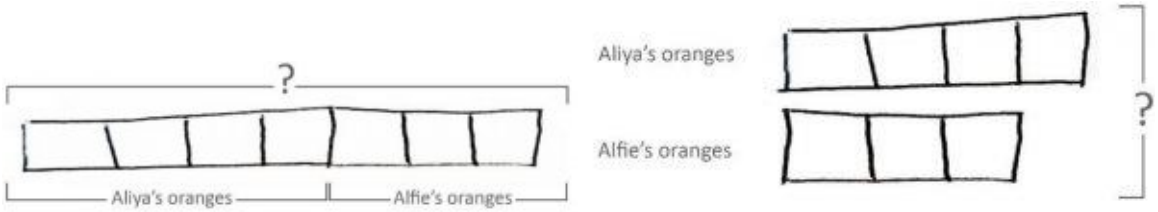
The next stage is to replace the 'real' objects with objects that represent what is being discussed (in this case, we replace the 'real' oranges with button counters):

The next stage is to move away from the concrete to the pictorial.

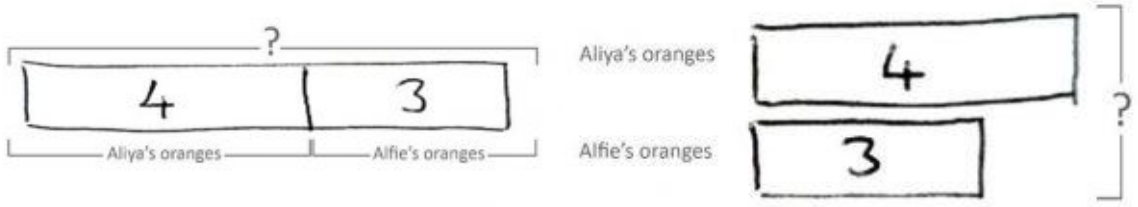
A general rule of thumb would be that towards the end of Year 1 or start of Year 2, pupils should be able to understand and represent simple addition (and subtraction) word problems pictorially and assign written labels in a bar model.



The penultimate stage is to represent each object as part of a bar, in preparation for the final stage:



The final stage stops the 1:1 representation. Each quantity is represented approximately as a rectangular bar:



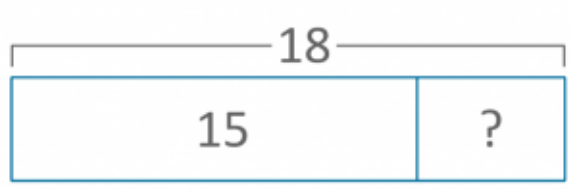
Subtraction

The same concrete to pictorial stages can be applied to subtraction. However, whereas with addition it is really down to the pupil's preference as to which of the 2 bar representations to use, with subtraction the teacher can nudge to pupils to one or other.

One represents a 'part-part-whole' model, the other a 'find the difference' model. Each will be more suited to different word problems and different pupils.

Part-part-whole

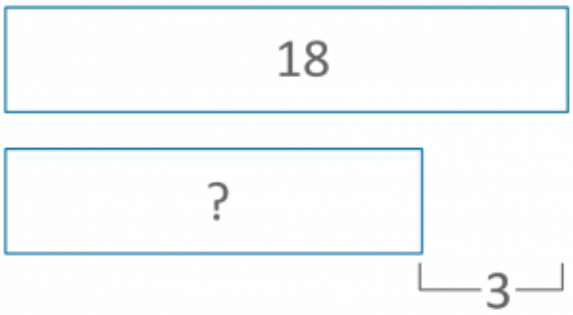
Austin has 18 lego bricks. He used 15 pieces to build a small car. How many pieces does he have left?



Calculation: 18 – 15 =

Find the difference

Austin has 18 lego bricks. Lionel has 3 lego bricks. How many more lego bricks does Austin have than Lionel?

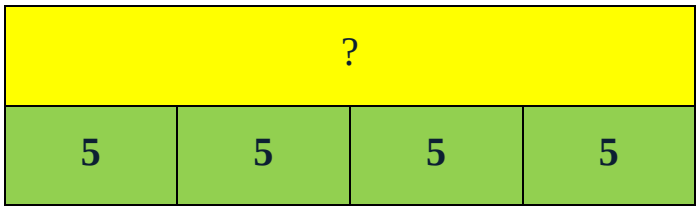


Calculation: 18 – 3 =

Multiplication

Bar models of multiplication start with the same 'real' and 'representative counters' stages as addition and subtraction. Then moves to its final stage, drawing rectangular bars to represent each group:

Each box contains 5 cookies. Lionel buys 4 boxes. How many cookies does Lionel have?



Division

Due to the complexity of division, it is recommended to remain grouping and sharing until the final stage of bar modelling is understood. Then word problems such as the 2 below can be introduced:

Sharing

Grace has 27 lollies. She wants to share them into 9 party bags for her friends. How many lollies will go into each party bag?

Grouping

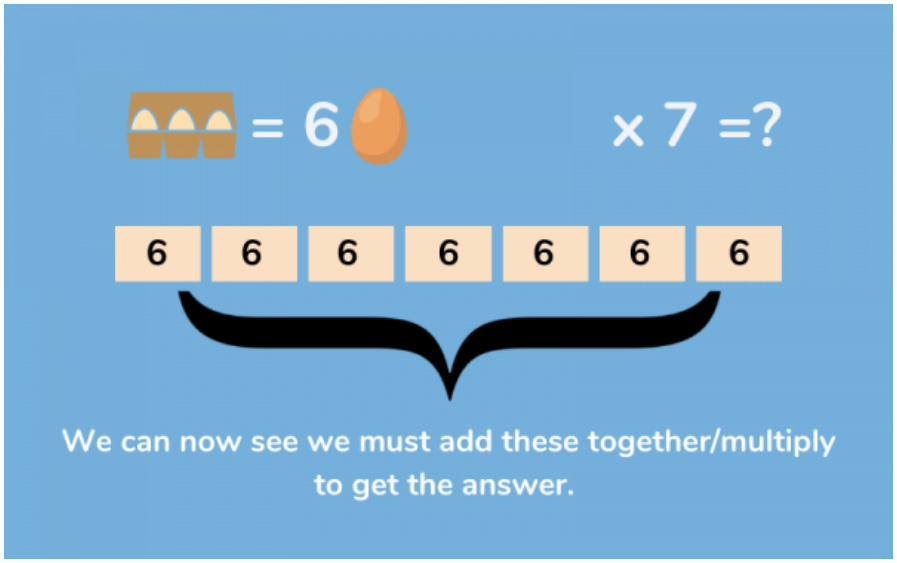
Grace has 27 lollies for her party friends. She wants each friend to have 3 lollies. How many friends can she invite to her party?

Progression in bar modelling on from KS1 to KS2

We can now teach pupils how to use the bar model for a deeper understanding of complex problems during Key Stage 2 and particularly in preparation for KS2 SATs.

The key question at any stage, at any age is: What do we know? By training pupils to ask this when presented with word problems themselves, they quickly become independent at drawing bar models.

For example, in the problem: Egg boxes can hold 6 eggs. We need to fill 7 boxes. How many eggs will we need?



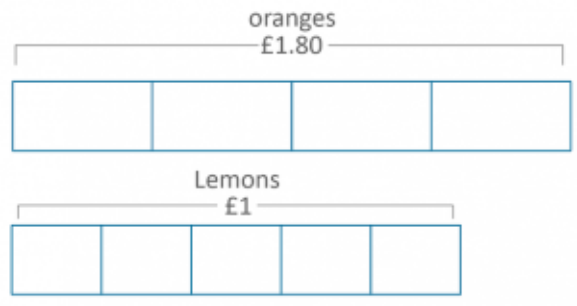
We know that there will be 7 egg boxes, so we know we can draw 7 rectangular bars. We know that each box holds 6 eggs, so we can write '6 eggs' or '6' in each of those 7 rectangular bar. We know we need to find the amount of eggs we have altogether. We can see we will need to use repeatedaddition or multiplication to solve the problem.

Four operations word problems

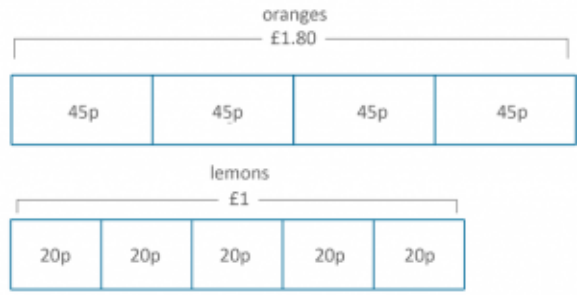
In a sample KS2 SATs, pupils are asked:

A bag of 5 lemons costs £1. A bag of 4 oranges costs £1.80. How much more does one orange cost than one lemon?

Pupils could represent this problem in the below bar model, simply by asking and answering 'what do we know?'



From here it should be straightforward for the pupils to 'see' or visualise their next step. Namely, dividing £1.80 by 4 and £1 by 5. Some pupils will not need the bar model to represent the next stage, but if they do, they would calculate and then allocate the cost onto the model:

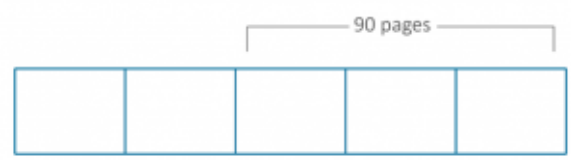


Then those pupils that needed this stage, should be able to see that to answer the question, they need to calculate 45p – 20p. With the answer of 25p.

Word problems with fractions

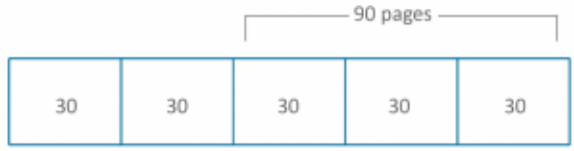
Here's an example:

On Saturday Lara read two fifths of her book. On Sunday, she read the other 90 pages to finish the book. How many pages are there in Lara's book? If we create our bar model for what we know:



Pupils will then see that they can divide 90 by 3:

As fractions are 'equal parts' – a concept they should be familiar with from key stage 1 – they know that the other 2 fifths (Saturday's reading) will be 30 pages each:



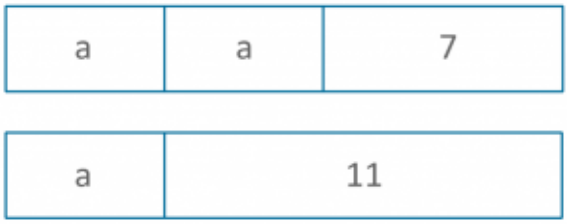
Then they can calculate 30 x 5 = 150

Equations with the bar model in KS2

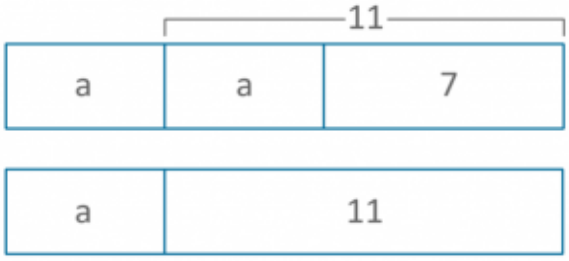
There are lots of other areas bar models can assist pupil's understanding such as ratio, percentages and equations. In this final example, we look at how an equation can be solved using the comparison model:

$2a + 7 = a + 11$

Let's draw what we know in a comparison model, as we know both sides of the equation will equal the same total:



The bars showing 7 and 11 could have been a lot smaller or larger as we don't know their relative value to 'a' at this stage. However, it is crucial that the 'a' appearing first in both bars is understood to be equal (even if it is only approximately equal when drawn freehand in the bar). This allows the pupil to 'see' that to work out the second 'a' in the top bar, they can calculate 11-7.



So if that 'a' is 4, then both the other 'a's will also be 4. So each side of the equation will total 15. The below model shows all sections completed. This is not necessary for the pupils to do, the representation is merely useful until they can see the steps necessary to calculate whatever they are faced with: